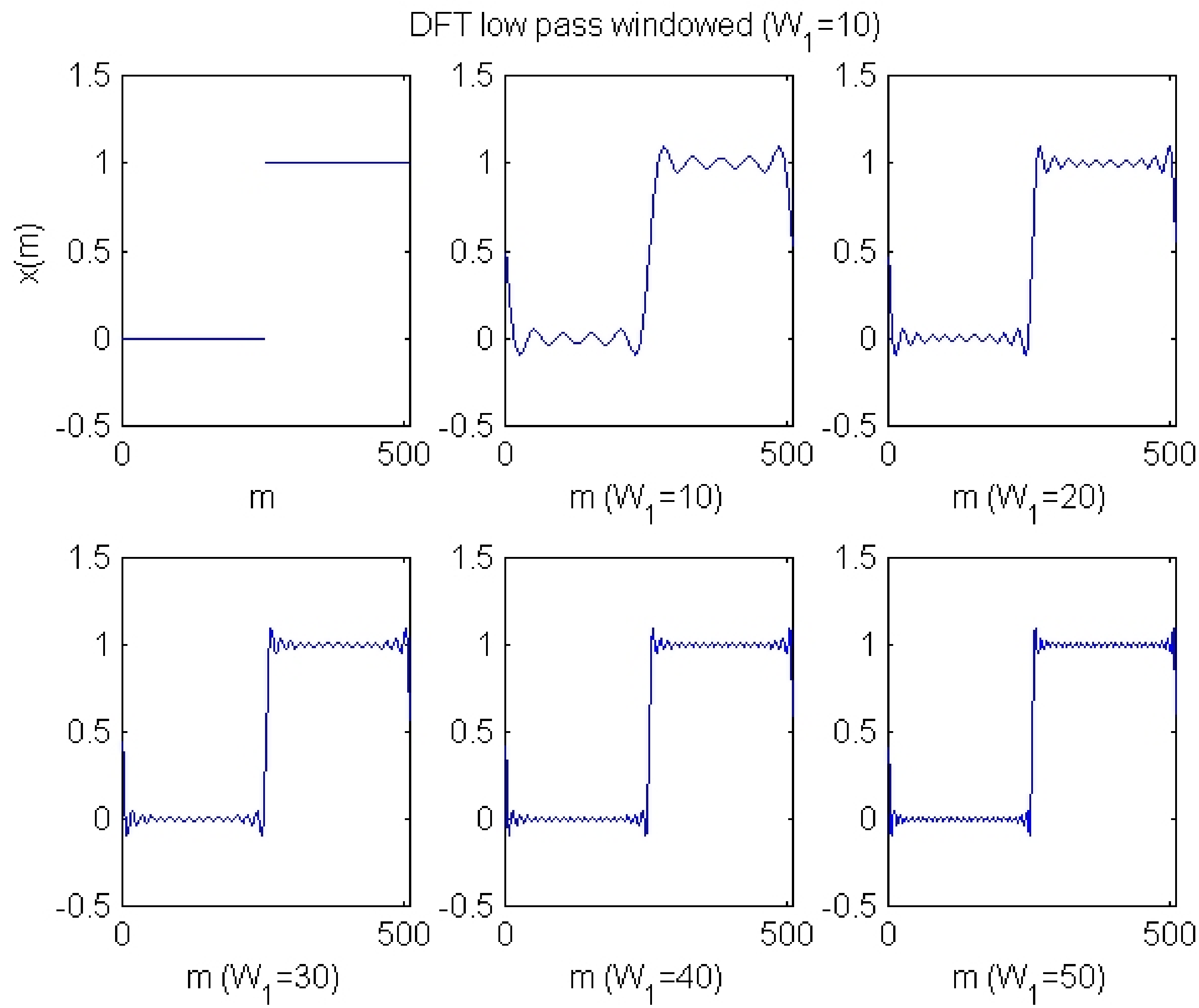
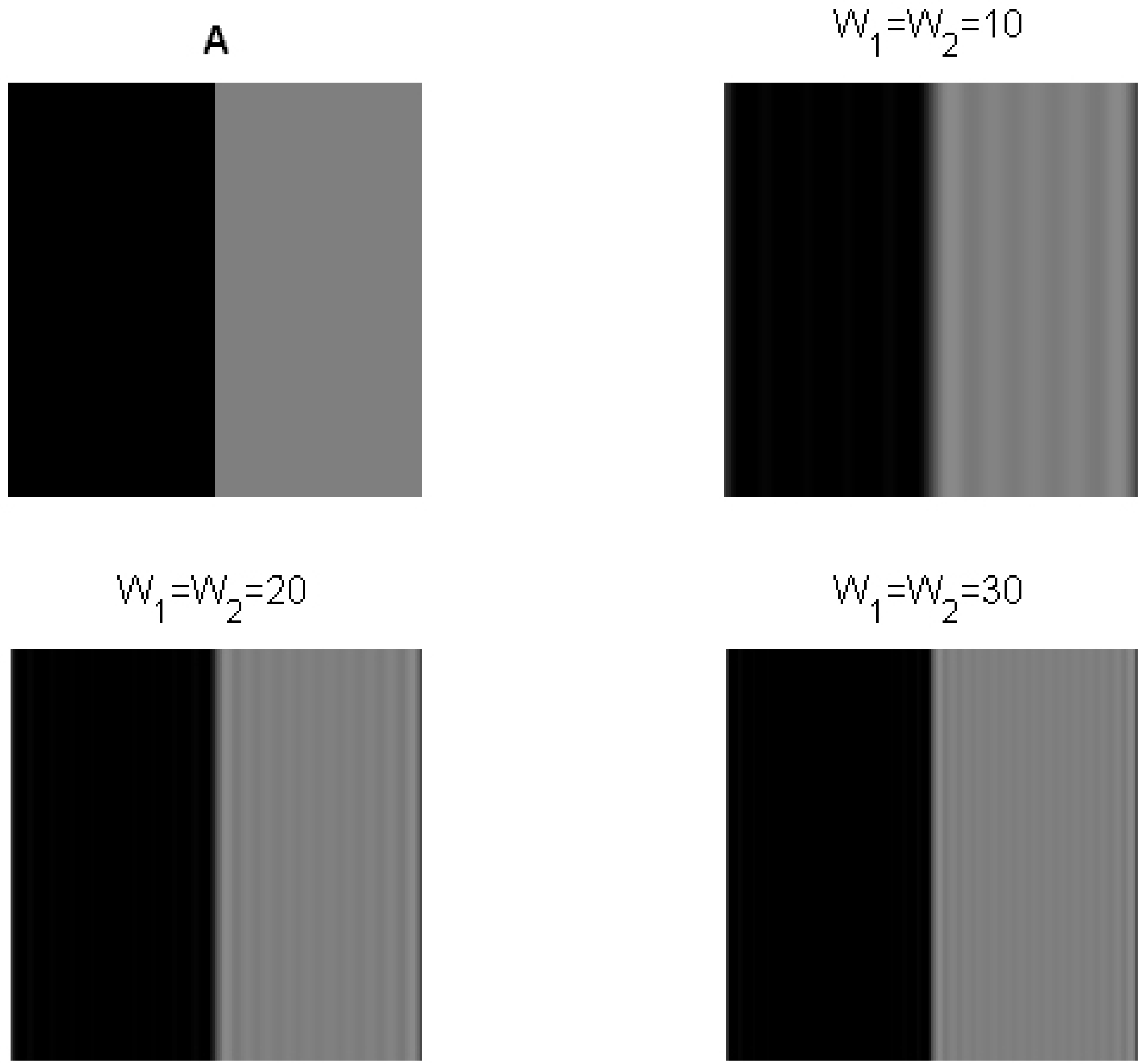
Summary of Lecture 8

- In lecture 8 we learnt how to low-pass, high-pass and band-pass filter images in two different ways.
- We considered two filtering applications:
  - Subsampling by low-pass antialiasing filters.
  - Noise reduction/removal by low-pass filters

Fourier Transforms and Gibbs Phenomenon



Fourier Transforms and Gibbs Phenomenon contd.

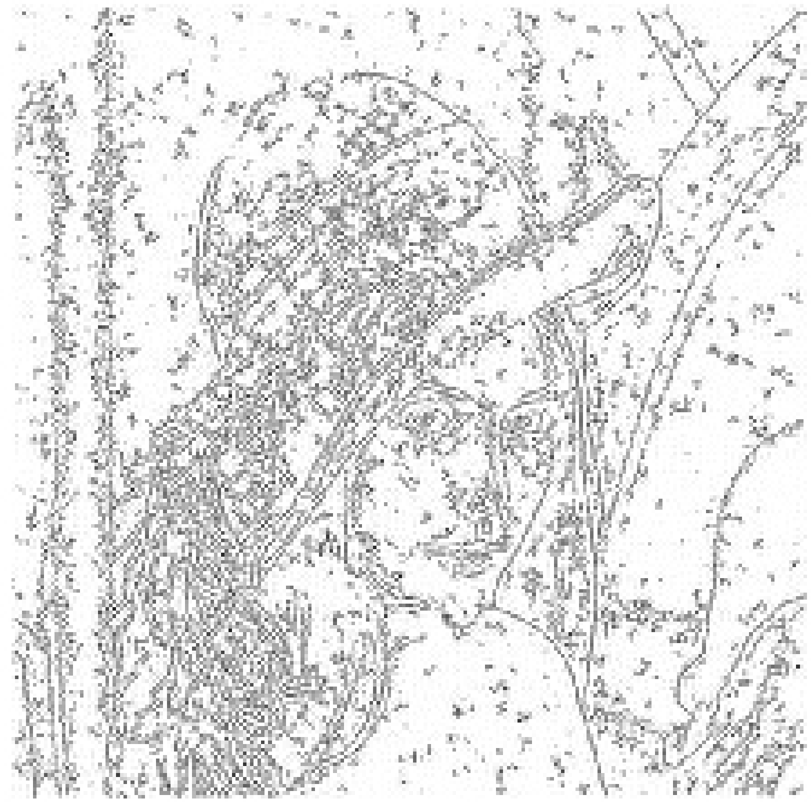


Images and Edges

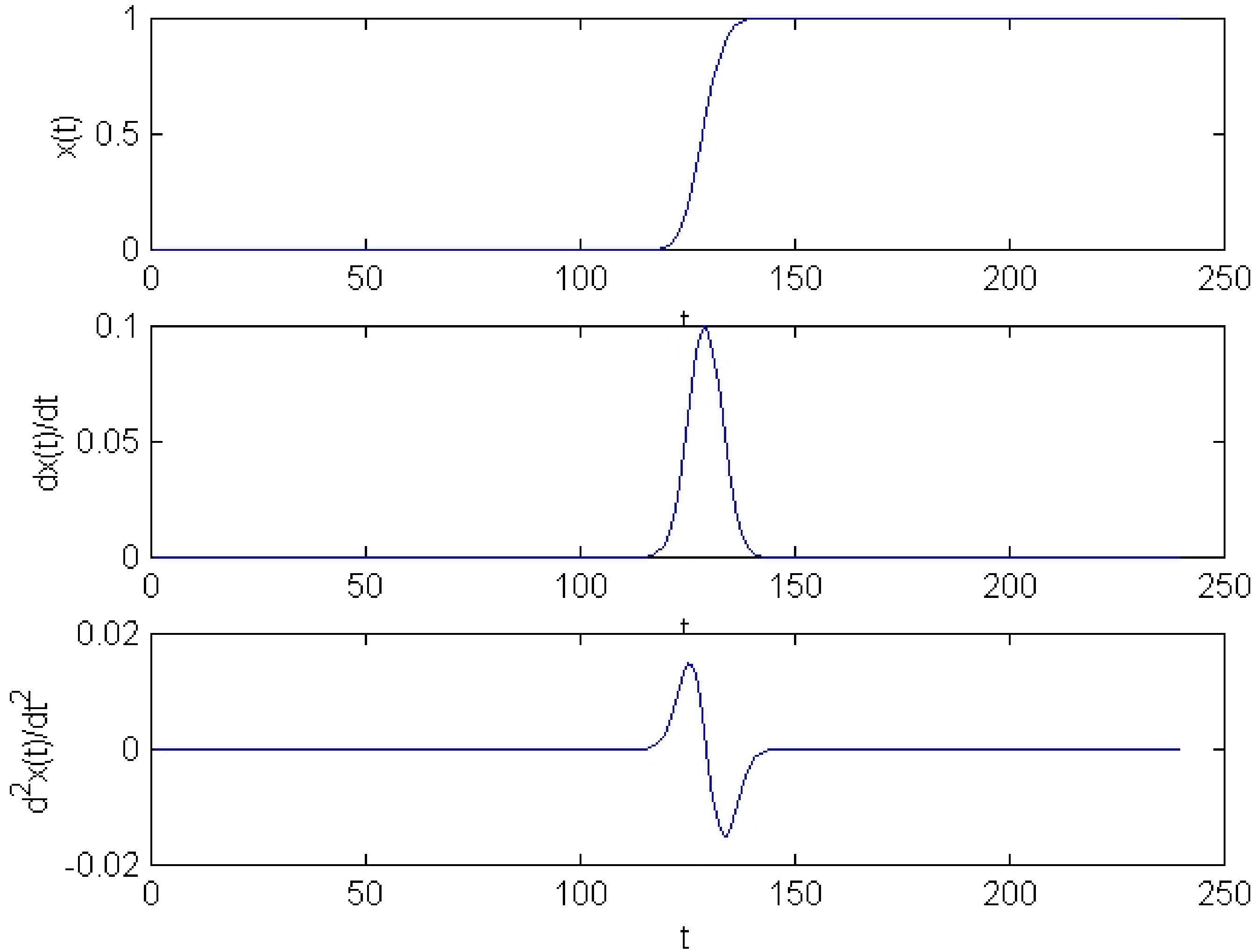
Images and Edges contd.

Edges and Noise

Noisy Image (A+10*randn(512))



Edge Detection - Motivation



Edge Detection Algorithms - Continuous Time

Suppose $x(t)$ is a continuous time signal containing "edges". The edges can be detected in two ways via:

1. Calculating $y(t) = \frac{dx(t)}{dt}$ and finding the locations t_i where $|y(t)|$ has a local maximum, i.e., there is an edge at location t_i if:

   $$|y(t_i)| - |y(t_i + \epsilon)| \geq 0 \ , \qquad \forall \ 0 < |\epsilon| << 1$$

   In order to incorporate "non-ideal" signals we will say that there is an edge at location t_i if $|y(t_i)| > T_e$ where $T_e > 0$ is a threshold or "the strength of the edge". However, we will still refer to this algorithm as detecting the local maxima.

Edge Detection Algorithms - contd.

2. Calculating $y(t) = \frac{d^2x(t)}{dt^2}$ and finding the locations t_i of the zero crossings of $y(t)$, i.e., there is an edge at location t_i if:

$$y(t_i + \epsilon) > 0 \Rightarrow y(t_i - \epsilon) < 0, \qquad \forall\ 0 < \epsilon << 1$$

or

$$y(t_i + \epsilon) < 0 \Rightarrow y(t_i - \epsilon) > 0, \qquad \forall\ 0 < \epsilon << 1$$

Let

$$c(t) = \begin{cases} y(t) & |y(t)| > T_e \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $T_e > 0$ is a threshold. In order to incorporate "non-ideal" signals we will say that there is an edge at location t_i if $c(t)$ has a zero crossing at t_i. We will still refer to this algorithm as detecting the zero crossings.

Edge Detection Algorithms - Discrete Time

- We can simply approximate the derivatives d/dt and d^2/dt^2 by discrete approximations. These approximations are called gradient operators.
- Images are two dimensional. If we only incorporate "row" gradient operators we can only detect vertical edges:
  - Do two passes: Detect vertical edges and then detect horizontal edges.
  - We can define "directionless" gradient operators that detect horizontal and vertical edges in one pass.

General Algorithm

General edge detection algorithm:

1. Apply gradient operator to the image $\mathbf{A}$.
2. Detect local maxima or zero crossings (with T_e).
3. Generate an "edge-map" $\mathbf{E}_A$ where

$$E_A(m,n) = \begin{cases} 255 & \text{pixel } (m,n) \text{ is an edge pixel} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

4. If a two pass approach is used to yield horizontal $\mathbf{E}_A^h$ and vertical $\mathbf{E}_A^v$ edge maps one can define an overall map by

$$E_A(m,n) = \begin{cases} 255 & E_A^h(m,n) = 255 \text{ or } E_A^v(m,n) = 255 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Gradient Operators

- Gradient operators are defined as filters and application of gradient operators is implemented by linear filtering of the image.
- Note however that edge detection involves a nonlinear step where we detect local maxima for "derivative" operators or zero crossings for "second derivative" operators on the linearly filtered output.

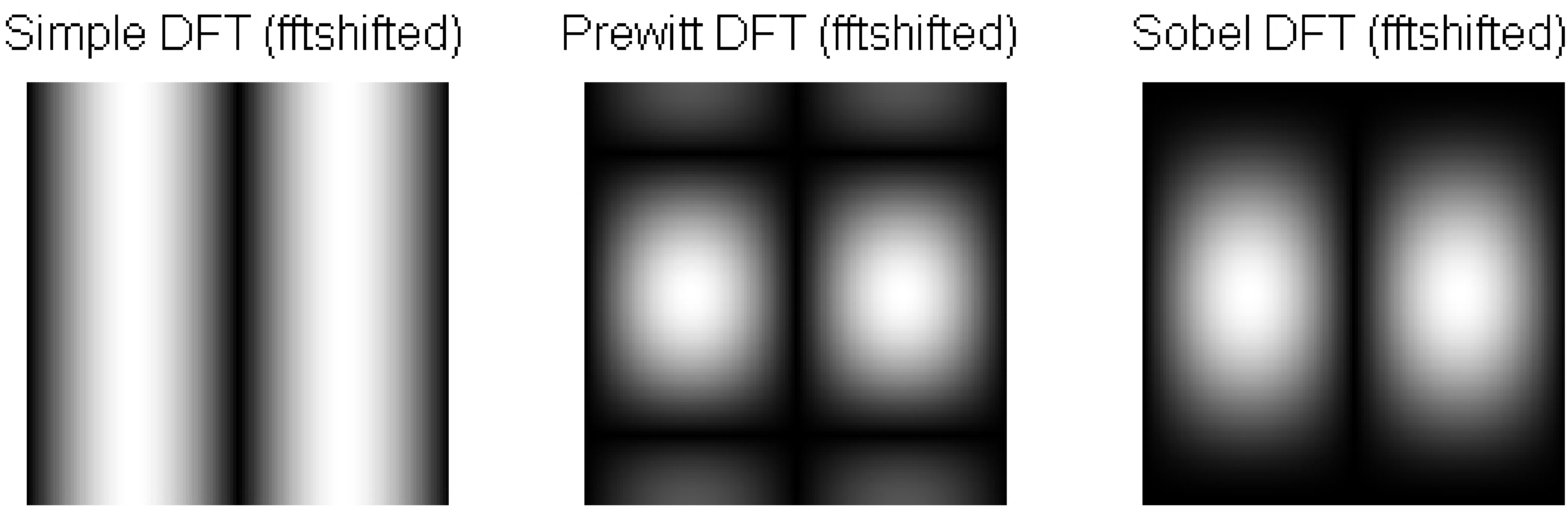
Gradient Operators contd.

- Local Maxima: Determining local maxima in two dimensions is established by comparing the absolute value of the filtered output at (m, n) to T_e.
- Zero Crossings: Let $\mathbf{B}$ denote the filtered and thresholded (with T_e) image.
  - If $B(m, n) = 0$ and $sign(B(m, n-1)B(m, n+1)) = -1$ then there is a vertical edge at pixel (m, n), etc.
  - If $B(m, n) \neq 0$ and $sign(B(m, n)B(m, n+1)) = -1$ then there is a vertical edge between pixels (m, n) and $(m, n+1)$.
- Zero Crossings for Directionless Operators: On a two dimensional grid, a zero-crossing is said to occur wherever there is a zero crossing in *at least* one direction.

First Derivative Operators

- Simple: $[-1 \quad 0 \quad 1]$, Prewitt: $\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$, Sobel: $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

Operators for horizontal edges can be obtained by transposing the filters.



Example - Vertical Edges

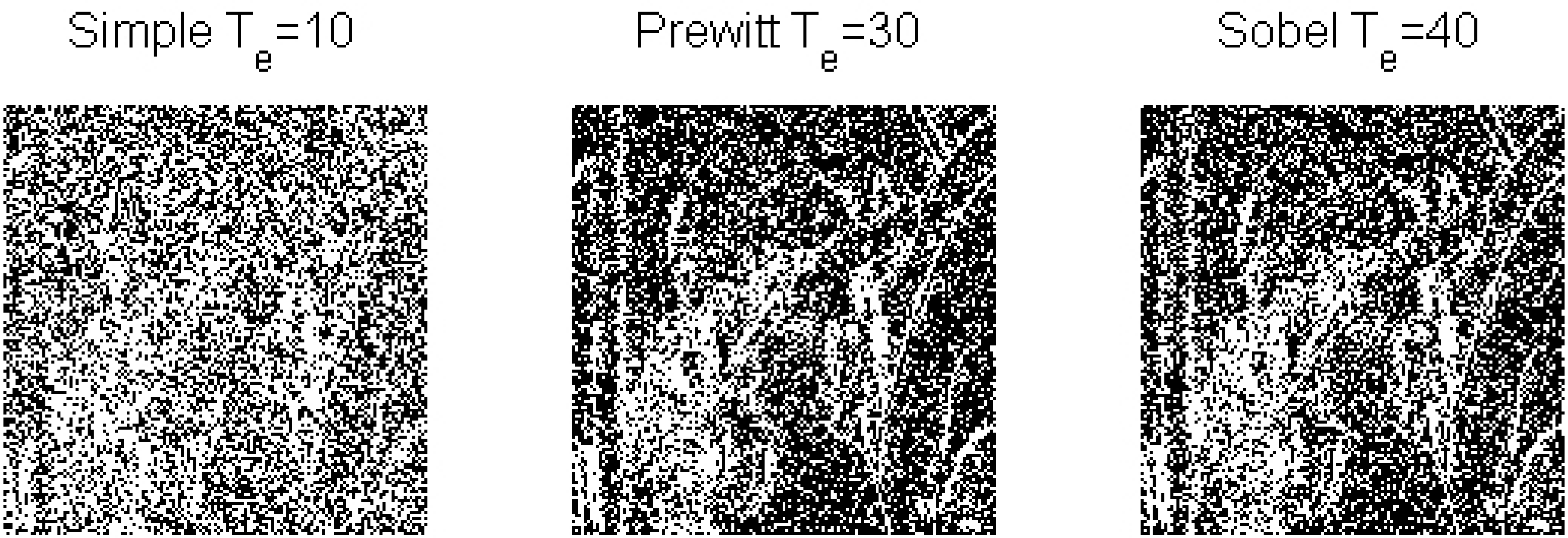
Simple T_e=10

Prewitt T_e=30

Sobel T_e=40

Detecting vertical edges on Lenna.

Example - Vertical Edges contd.

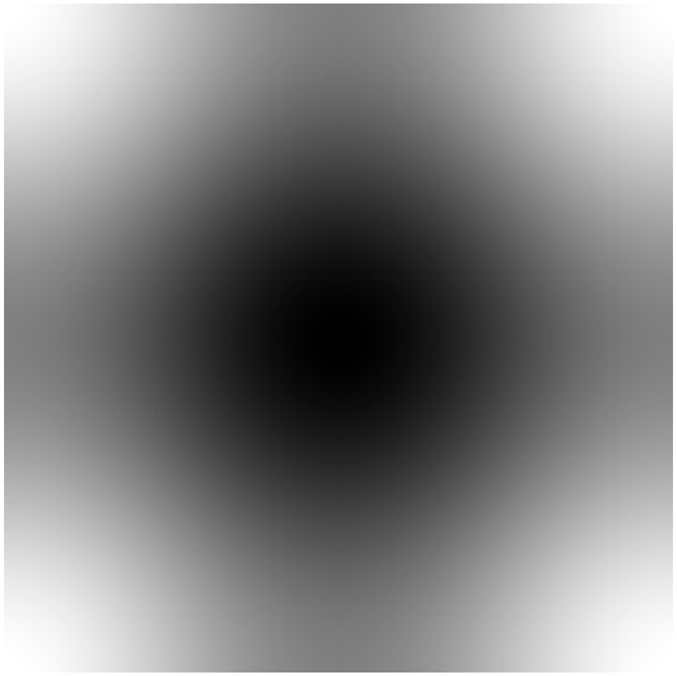


Detecting vertical edges on Lenna+10*randn(512).

Directionless Second Derivative Operators

- It can be shown that the Laplacian Operator $\nabla^2 f = \partial^2 f / \partial^2 x + \partial^2 f / \partial^2 y$ is rotationally symmetric.
- Hence we can expect a discrete approximation to this operator to be approximately rotationally symmetric.
- One such approximation is $Z(m,n) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

DFT of Discrete Laplacian (fftshifted)



Edge Detection and Noise

- The derivative operators $D(m,n)$ used in edge detection are susceptible to noise.
- One approach would be to low pass filter ($L(m,n)$) an image first (to reduce noise) and then employ the derivative operator.
- In practice one chooses spatial low pass filters in order to avoid the Gibbs phenomenon.
- Let $A(m,n)$ denote the original image. Then we have $\mathbf{C} = \mathbf{D} \otimes \mathbf{L} \otimes \mathbf{A}$ as the overall filtered image on which nonlinear decisions can be made.
- One can thus define a combined gradient operator $\mathbf{H} = \mathbf{D} \otimes \mathbf{L}$.

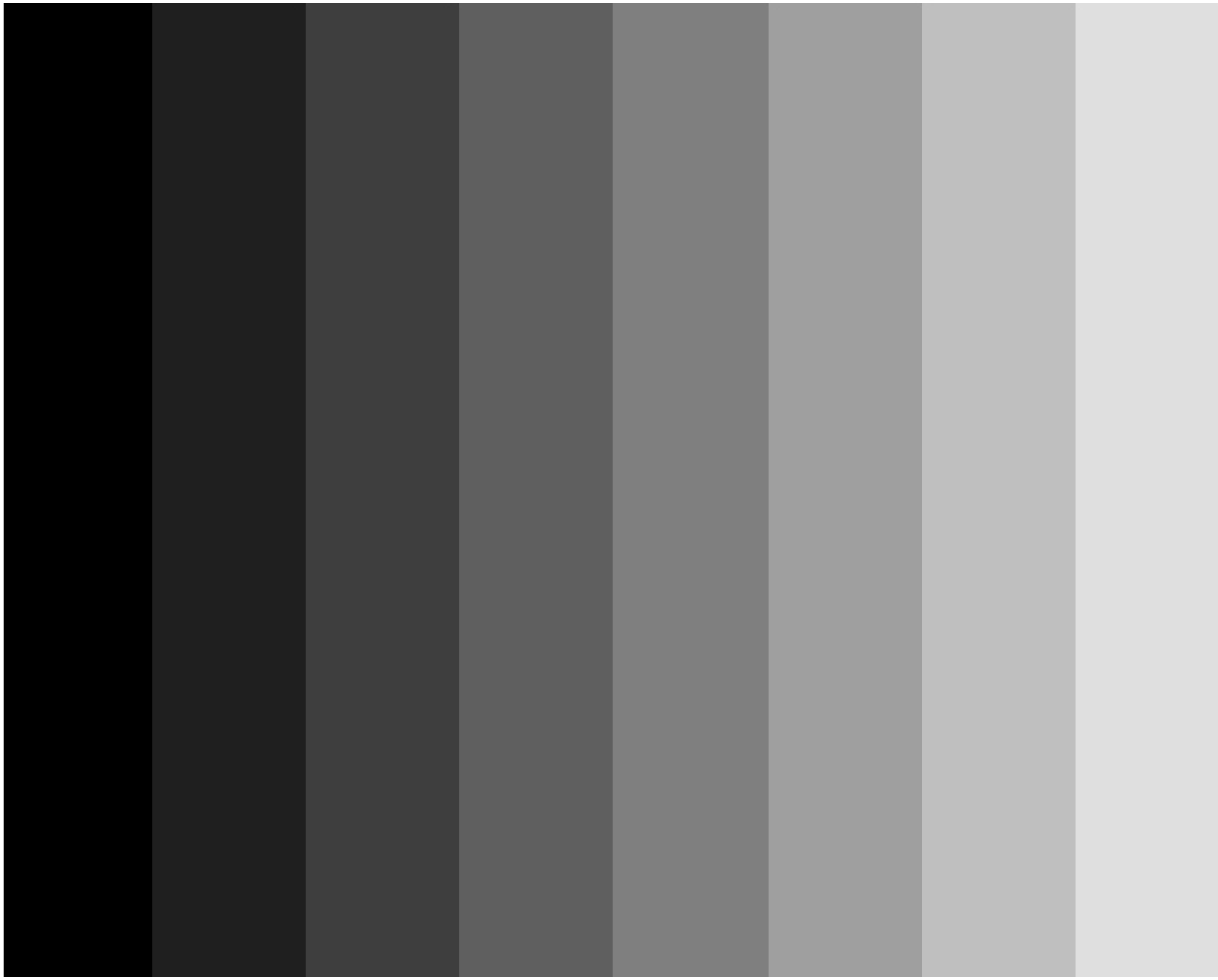
The Laplacian of a Gaussian Gradient Operator

- The Gaussian sequence defined by $L(m,n) = e^{-(m^2+n^2)/2\sigma^2}$ is a low-pass sequence and can be used as a low-pass filter (σ controls the bandwidth of this filter).
- The Laplacian of a Gaussian $\mathbf{H} = \mathbf{Z} \otimes \mathbf{L}$ can thus be defined as a second derivative gradient operator with favorable performance under noisy conditions.
- σ is chosen large for very noisy conditions and small for relatively noiseless conditions.
- Typically $L(m,n)$ is employed as a finite extent sequence $P_1 \times P_2$ (m, n must be centered around zero $-1 \leq m, n \leq 1$ etc.). (Large values of σ will in general require larger extents.)
- Edges are detected using the zero crossings of the filtered output (with T_e).

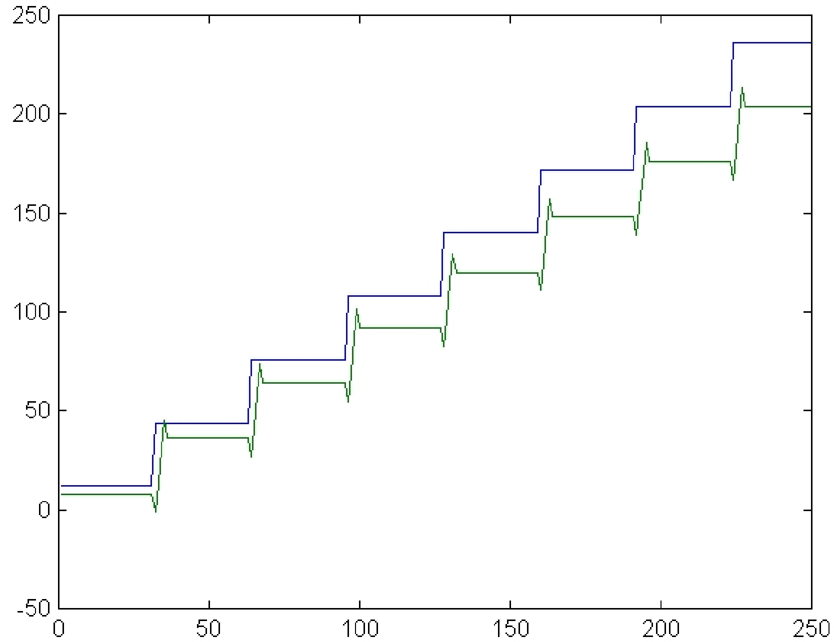
Example

$P_1 = P_2 = 3, T_e = 30, \sigma = 2$

Human Visual System and Mach Bands



Low-pass Response of the Human Visual System



- The human visual system low-pass filters the scenes under observation.
- We will exploit this observation by "hiding" processing errors where humans cannot see them.

Summary

- In this lecture we learnt about the Gibbs phenomenon.
- We learnt edge detection with:
  - Gradient operators based on first derivatives and associated local maxima.
  - Gradient operators based on second derivatives and associated zero crossings.

  (please read pages 347-353 from the textbook).
- We learnt about Mach bands and the low-pass nature of the human visual system (please read pages 51-56 from the textbook).

Homework IX

1. Detect horizontal and vertical edges in your image using the simple, Prewitt and Sobel operators. Show horizontal edge maps, vertical edge maps and combined edge maps. Comment on differences between horizontal and vertical edge maps. Experiment with the relevant thresholds and try to choose them as best as you can.

2. Detect edges in your image by using the zero crossings of the Laplacian of Gaussian operator. Adjust P_1, P_2, T_e, σ to obtain the best results.

3. Repeat 2 by starting with a noisy version of your image (say with image+10randn()). Try to adjust the parameters to match noiseless performance. Comment on the differences of the result with 2.

4. Generate a one dimensional signal and its perceived version as I have done. Try to find a filter which when convolved with the original signal gives the "perceived" signal. (Hint: Use DFTs of the two signals and use the convolution → multiplication property. Be wary of divisions by zeros or small values.) Is the filter you have discovered low-pass? Show the 1D DFT and the filter itself.

References

[1] A. K. Jain, *Fundamentals of Digital Image Processing.* Englewood Cliffs, NJ: Prentice Hall, 1989.